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DISCUSSION OF PROCEEDINGS - SEPARATES

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HYDRAULICS DIVISION

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Discussion of "BACKWATER FUNCTIONS BY NUMERICAL INTEGRATION"

by Clint J. Keifer and Henry Hsieng Chu
(Proc. Sep. 383)

FRANCIS F. ESCOFFIER,[6] A. M. ASCE.—The computation of water-surface profiles in circular conduits has always been difficult and laborious. The authors have prepared tables of functions that will greatly facilitate these computations. However, the use of these functions in the manner outlined in the paper involves the disadvantage that it is necessary to begin with a known or assumed depth and to solve for the reach length. Usually, in engineering practice, the length is given and the depth is to be computed, and therefore several trials are ordinarily required before a solution can be obtained. Such a disadvantage is readily overcome by means of the graphical construction shown in Fig. 7. In this diagram, the function Ψ has been plotted as the ordinate. The function Φ is the abscissa in the left-hand part of the diagram, and the depth variable $J = y/D$ is the abscissa in the right-hand part.

[6] Hydr. Engr., Corps of Engrs., U. S. Dept. of the Army, Mobile, Ala.

The graphical solution included in the diagram is that for the drop-down curve shown in Fig. 8. To understand the solution it is only necessary to consider that, if Eq. 12 is multiplied through by S_0/D, there is obtained the result—

$$I = \Phi_2 - \Phi_1 - s\,(\Psi_2 - \Psi_1) \quad \ldots\ldots\ldots\ldots\ldots\ldots (19a)$$

in which

$$I = \frac{S_0\, x_{1\to 2}}{D} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19b)$$

and

$$s = \frac{Q^2}{D^5} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (19c)$$

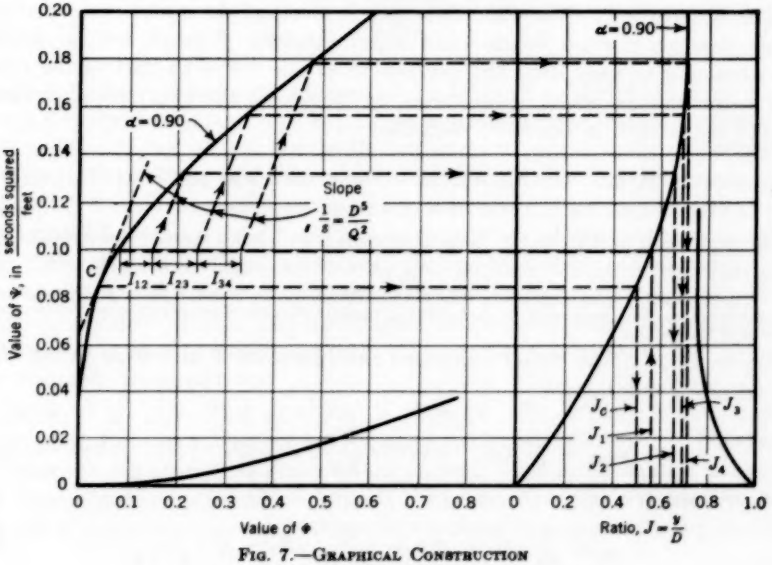

FIG. 7.—GRAPHICAL CONSTRUCTION

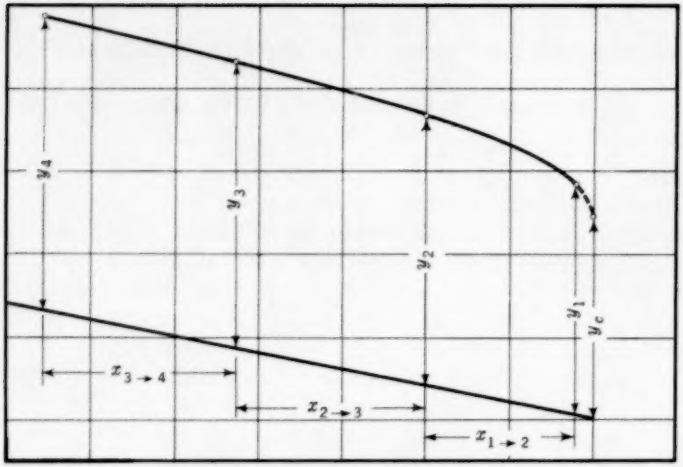

FIG. 8.—DROP-DOWN CURVE

For the purposes of this discussion, Eq. 19*a* is rearranged in the form:

$$I = (\Phi_2 - s\,\Psi_2) - (\Phi_1 - s\,\Psi_1) \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

A given value of $\Phi - s\,\Psi$ appears in the left-hand part of the diagram as a line having a slope $1/s$; Eq. 20 indicates that the horizontal distance between the two successive lines there represented is equal to I. It is believed that the procedure is self-explanatory. Critical depth occurs at the point where the tangent to the curve has a slope equal to $1/s$.

The statement made by the authors (under the heading, "Numerical Integration. Equation for Nonuniform Steady Flow") that the backwater functions Φ and Ψ cannot be used in a conduit with a slope equal to the critical slope is not correct. The only difficulty is that, as the water-surface profile passes through normal depth (and simultaneously through critical depth), it is necessary to pass from the asymptote of one curve to that of the other. This inconvenience could be liminated by displacing the two curves vertically so as to make their asymptotes coincide.

HENRY J. MILES,[7] A. M. ASCE.—The writer was particularly interested in the first part of the paper which is concerned with numerical integration, as he had used a similar method in a part of his doctoral dissertation.[8] Referring to Eq. 12, the writer expressed the functions Φ and Ψ in terms of S_F instead of S_0, obtaining:

[7] Prof. of Hydr. Eng., Agri. and Mech. College of Texas, College Station, Tex.

[8] "New Methods of Analyzing Steady Flow in Open Channels," by Henry J. Miles, thesis presented to Rutgers University, New Brunswick, N. J., in 1950, in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

$$x_{1 \to 2} = \frac{D}{S_F}(A_2 - A_1) - \frac{V^2}{2\,g}\,(\text{full})\,(B_2 - B_1) \ldots\ldots\ldots (21a)$$

which reduces to

$$x_{1 \to 2} = \frac{D}{S_F}(A_2 - A_1) - \frac{Q^2}{S_F\,D^4}(B_2 - B_1)\,\frac{16}{\pi^2\,2\,g} \ldots\ldots\ldots (21b)$$

in which $A = f_1\left(\frac{y}{D}, \frac{S_0}{S_F}\right)$ and $B = f_2\left(\frac{y}{D}, \frac{S_0}{S_F}, \frac{V^2/2g}{V^2\,(\text{full})/2g}\right)$, which are the reciprocals of those functions shown in Fig. 5. As far as could be determined when the writer first presented these ratios, their use was novel. The authors' statement (under the heading, "Backwater Curves in Uniform Channels by the Step Method") that they had "devised two new approaches" was an interesting confirmation of the writer's opinion.

After developing this method for circular pipes, the writer applied it also to rectangular channels. This procedure was supplemented by a graphical solution to aid in the application of the backwater function.

The authors are to be complimented for making a valuable tool available to the engineering profession. The writer can appreciate from his own experience the immense volume of work involved in preparing a table of functions as complete as the unabridged versions of the authors' backwater functions.

RICHARD SILVESTER.[9]—Graphs and a nomograph which are helpful in computing water-surface profiles with nonuniform flow for various channel shapes, including circular channels and U-shaped channels, have been published elsewhere by the writer.[10,11] Values of $K = 1.486\,AR^{\frac{2}{3}}$ were graphed against depth, y, for certain channel shapes with various bottom widths. Similarly, values of $M = \sqrt{A^3/b_w}$ were plotted on the same graphs for substitution in the nonuniform equation,

[9] Lecturer in Civ. Eng., The Univ. of Western Australia, Nedlands, Australia.

[10] "Flow in Open Channels," by Richard Silvester, *Journal*, Inst. of Engrs., Australia, Vol. 25, 1953, p. 155.

[11] "New Graphs Aid Open Channel Calculations," by Richard Silvester, *Water Power*, September, 1954, p. 338.

$$\frac{dy}{dx} = \frac{S_0 - \left(\frac{Q\,n}{K}\right)^2}{1 - \frac{Q^2}{g\,M^2}} = \frac{S_0 - S}{1 - S_v} \quad \ldots\ldots\ldots\ldots\ldots\ldots (22)$$

in which S_v is the velocity component $Q^2/(g\,M^2)$. Eq. 22 and Eq. 3 are similar except for sign convention. In Eq. 22, dx is positive measured downstream and dy is positive for an increase in depth downstream (negative for a decrease).

The graph of K and M for circular channels and U-shaped channels is shown in Fig. 9. Numerals on the curves of Fig. 9 indicate U-values and D-values, in feet. Solution of the terms $(Q\,n/K)^2$ and $Q^2/(g\,M^2)$ is facilitated by a nomograph (Fig. 10). Figs. 9 and 10 are intended for use with step computations. It is recommended that, when the limits of the profile have been determined, the stage variation be divided into a number of increments which are variable, the smallest being near the critical depth. The writer's solution of the "Illustrative Example" is as follows:

As $M_c = Q/\sqrt{g} = 28.2$, $y_c = 3.45$ ft (Fig. 9). From Table 6 (commencing the computations at a depth of 3.6 ft), the total length is 3,841 ft. This resulting length is less than that given by the authors. By using smaller increments in y, this discrepancy would be reduced. The negative sign obtained for dy/dx in Table 6 indicates a decrease in depth downstream as required by the M_2-curve.

TABLE 6.—BACKWATER COMPUTATIONS

Depth, y, in feet	Increment, dy, in feet	Conveyance, K	M	Slope, S	Slope, S_v	$S_o - S$	$-\frac{dy}{dx}$	Increment, dx, in feet
3.6	0.1	36.3	31	0.0033	0.89	0.0023	0.0209	5
3.7	0.1	38	33	0.003	0.82	0.002	0.0111	9
3.8	0.1	39.7	35	0.0027	0.7	0.0017	0.00567	35
4.0	0.2	44	38	0.0023	0.62	0.0013	0.00342	59
4.2	0.2	46	41	0.0021	0.51	0.0011	0.00225	89
4.4	0.2	49	45	0.0018	0.43	0.0008	0.0014	143
4.6	0.2	51	49	0.00163	0.38	0.00063	0.00102	196
4.8	0.2	54	54	0.00147	0.31	0.00047	0.00068	294
5.0	0.25	57	60	0.0013	0.24	0.0003	0.000395	633
5.25	0.25	58	69	0.00128	0.19	0.00028	0.000346	722
5.5	0.25	59	78	0.00125	0.15	0.00025	0.000294	850
5.75	0.25	58	97	0.00128	0.095	0.00028	0.00031	806

Although the writer's procedure may require a little more time than substitutions into the authors' graphs and tables, the former gives a better over-all representation of the profile. One can see at a glance the depth at any distance along the curve. The long distances occurring near the normal depth for small increases or decreases of depth are at once apparent; results for design purposes are thus more easily modified.

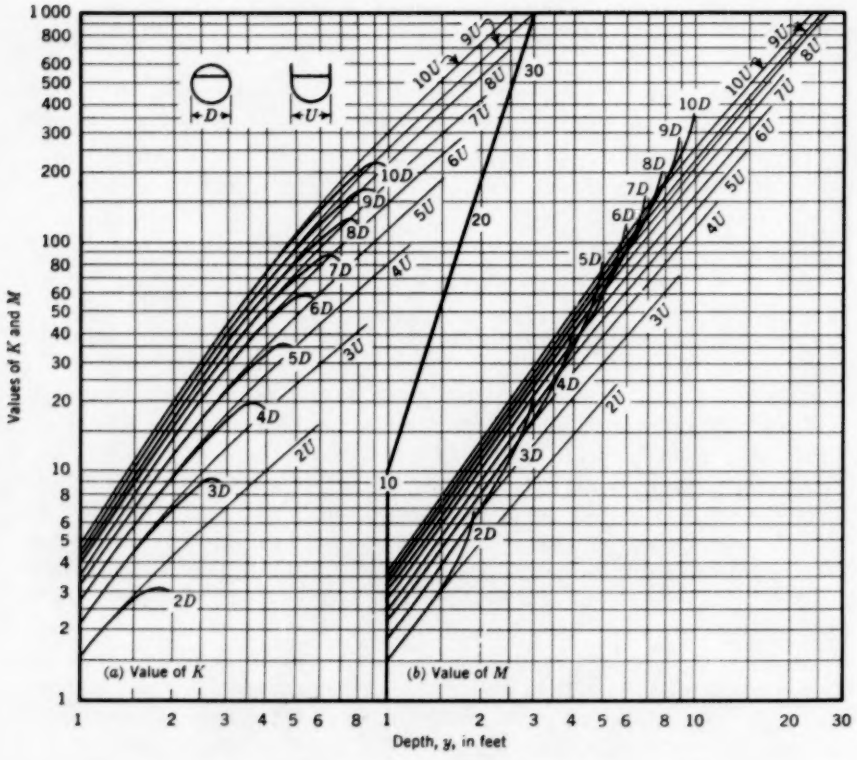

FIG. 9.—VALUES OF K AND M

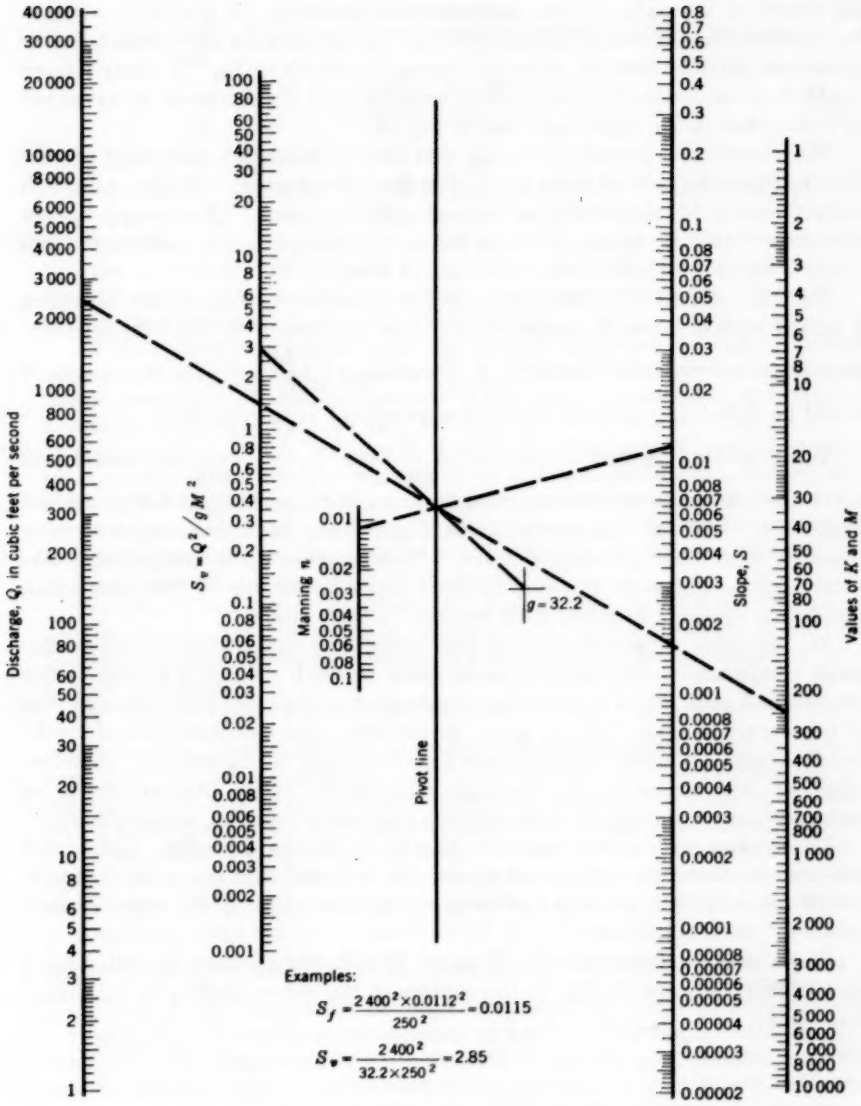

FIG. 10.—NOMOGRAPH FOR BACKWATER CURVES

CLINT J. KEIFER,[12] A. M. ASCE, AND HENRY HSIEN CHU.[13]—Mr. Escoffier has extended the value of the backwater functions by his graphical solution for determining the water-surface profile. It is true that the trial-and-error procedure must be used to solve for change in depth in Eq. 12 when a fixed length is given because the dependent variable y/D (in this case) is contained in both terms on the right-hand side of Eq. 12.

[12] Asst. Engr. of Sewer Design, Dept of Public Works, Bureau of Eng., Chicago, Ill.
[13] Senior Asst. Sewer Designer, Dept. of Public Works, Bureau of Eng., Chicago, Ill.

Mr. Escoffier is correct in stating that the functions are also valid for the case in which the bottom slope is equal to the critical slope. In the paragraph containing Eq. 14, the second sentence should have read—"The usage of these functions, however, must be restricted to conduits having positive bottom slopes (downward slopes in the direction of flow)."

Eq. 21*b*, presented by Mr. Miles, is indeed similar to Eq. 12; the functions A and B replace Φ and Ψ, respectively. The function B in Eq. 21*b*, however, contains three variables—namely, $\frac{y}{D}$, $\frac{S_0}{S_F}$, and $\frac{V^2/2g}{V^2\ (\text{full})\ /2g}$. In this form it would be difficult to present B in either graphical or tabular form.

When plotted against $\frac{y}{D}$, the ratios $\frac{S}{S_F}$ and $\frac{V^2/2g}{V^2\ (\text{full})\ /2g}$ are useful and convenient in backwater computation as shown in the writers' first step method (Table 5). Full-size charts similar to Fig. 5 have been prepared for many generally used closed conduit sections. Their application in computing backwater curves, whenever the step method seems preferable to the integration method, is accepted as a standard routine by the writers.

Mr. Silvester presented another step method by using a graph and nomograph comparable to the writers' second step method. Fig. 9 is similar to the left half of Fig. 6, and Fig. 10 is the counterpart of the right half although it is of nomographic form. Many tools to facilitate the computations of backwater curves by the step method have been published in engineering literature. Engineers may choose among them, guided probably by the computation to be performed, the accuracy desired, and the engineer's habitual acquaintance.

The writers wish to express their thanks to Messrs. Escoffier, Miles, and Silvester for their interesting comments. It is hoped that the paper has contributed some additional information on the determination of the water-surface profile in closed conduits.

Corrections for Transactions.—Page 10, line 16, the number, 19.960, should read 19,960; on page 13, Fig. 5, the caption of the curve reading h_F/h_w should read h_F/h_v.

Discussion of
"BACKWATER EFFECTS OF OPEN CHANNEL CONSTRICTIONS"

by H. J. Tracy and R. W. Carter
(Proc. Sep. 413)

H. J. TRACY[1] and R. W. CARTER,[2] A.M. ASCE—Professor Henry has provided a thoughtful and stimulating discussion by adapting the momentum and energy statements of equations 4 and 5 to arrive at a direct theoretical solution for the water level changes at sections (1) and (3). He has shown that this solution may be modified to include the measured friction effects to permit its application to non-frictionless flow.

As Professor Henry points out, section 4 is no longer the control section when the flow in the contracted stream is critical. The water surface elevation at section (1) is then independent of the reach downstream from section 3, and its determination may proceed from the recognition of this fact.

It is pertinent at this point to note again that the scope of the laboratory investigation was limited to tranquil flow; and that, in addition, the Froude number that has been used has been defined in terms of the total flow area within the constriction, and hence is meaningless as a critical flow criterion. It should also be noted that a reliable definition of critical flow for any channel other than rectangular is not known.

The direct solution proposed by Mr. Izzard may also be used if an approximation for the backwater is sufficient for the requirements of the user. The mean-depth relationship becomes less exact for channel shapes which are not rectangular.

Mr. Izzard's comparison of the magnitude of $h_{f_{1.3}}$ with respect to the corresponding size of $\alpha V_1^2/2g$ as a test of the applicability of the backwater ratio does not seem warranted to the writers. The laboratory tests led to the conclusion that at an n of about 0.050 the limit of the change in the backwater ratio due to channel roughness has been reached. If this conclusion is valid, the relative size of the two terms is unimportant. However, an important point has been brought out. In the application of the results of this study to the computation of the backwater at a specific constriction, the use of equation 1 requires the computation of $h_{f_{1.3}}$ from the Manning formula (in the laboratory, both h_1^* and Δh were measured directly). If $h_{f_{1.3}}$ is a large part of Δh, the results obtained from equation 1 may be approximate.

The writers disagree with the transformation of the Nagler formula to conform to the notation of this paper made by Mr. Hodges. Nagler's formula may be derived from the energy equation; assuming a frictionless rectangular channel and approximating the constricted depth at section (3) by the depth at section (4) minus some multiple, θ, of the velocity head at section (4). This requires Nagler's H_3 (5) to be the total fall, Δh, between sections (1) and (3), and not h_1^* as shown in the adaptation. The writers were unable

1. Hydr. Engr., Geological Survey, U. S. Dept. of the Interior.
2. Hydr. Engr., Geological Survey, U. S. Dept. of the Interior, Atlanta, Ga.

to check the computations shown in Mr. Hodges' table.

The coefficient θ defined by Yarnell[5] for use in the Nagler formula appears to be unrealistic for any except very small degrees of contraction. Using the laboratory data of this study to compute values of θ, this coefficient was found to vary primarily with $\underline{m}$, ranging from zero at an $\underline{m} = 0$ to a value in excess of 10.0 at $\underline{m} = 0.80$ (Yarnell's value; a constant 0.3).

In this study, the coefficient C has been used as an index to describe the geometry of the constriction in order to relate the change in constriction geometry to the corresponding variation in the backwater ratio. Mr. Hodges questions the conclusion that the backwater ratio can be independent of the length-width ratio ($\frac{L}{b}$) of the constriction, since C varies with $\frac{L}{b}$. The invariance of the backwater ratio with $\frac{L}{b}$ simply indicates that the ratio of h_1^* to Δh (although both vary with $\frac{L}{b}$) remains reasonably constant as $\frac{L}{b}$ is independently varied. This, however, is not characteristic of the effect of other geometry features. Thus, as the constriction parameters (other than $\frac{L}{b}$) varied, $\frac{h_1^*}{\Delta h}$ was found to vary. This variation was related to the coefficient C, and the results are shown in figure 7.

The writers wish to express their appreciation for the interest shown by Messrs. Henry, Izzard, and Hodges in presenting their alternate solutions and their comparisons with other data.

REFERENCES

5. Woodward, S. M., and Posey, C. J., "Hydraulics of Steady Flow in Open Channels," Chapter X. p. 127; John Wiley and Sons, Inc., 1941.

Discussion of
"MORNING-GLORY SHAFT SPILLWAYS: PROTOTYPE BEHAVIOR"

by Joseph N. Bradley
(Proc. Sep. 431)

JOSEPH N. BRADLEY[1]—Corrections—In Proceedings-Separate 538 the discussion by F. W. Blaisdell which was contained in the section devoted to the discussion of this paper (431) was actually a discussion of Proceedings-Separate 433 entitled "Morning-Glory Shaft Spillways: Performance Tests on Prototype and Model" by A. J. Peterka. Mr. Blaisdell's discussion of 431 appeared in the section of 538 that was devoted to the discussion of 433.

The writer is indebted to the several men who so ably discussed the paper, and is grateful for the additional information presented on morning-glory spillway operation. It was informative to learn from Mr. Donclon that an actual underwater inspection was made of the vertical bend and adjoining horizontal tunnel at the Kingsley Dam Spillway. In the majority of cases inspection has been made by probing with poles or pipes.

Mr. Walter's enlightening presentation and photographs of the spillway operation at Hungry Horse Dam, which occurred in 1954, is of particular interest in that this is the largest spillway of this type. The spillway operated for the better part of two days. Attention is called to Mr. Walter's discussion concerning the precautions taken in construction of the spillway shaft and vertical bend regarding elimination of construction joints in the bend, elimination of offsets throughout by grinding, and the final sandblasting and finish treatment. It is anticipated that these precautionary measures will, in the long run, materially reduce maintenance costs.

Inspection of the tunnel and vertical bend by probing, made subsequent to the writing of the discussion by Mr. Walter, showed no noticeable erosion of the concrete surfaces. Pressures measured in the throat section of the spillway during operation showed close agreement with those observed on the model, thus air demand to the prototype was also comparable to that in the model. The lowest pressure measured was 8 feet of water below atmospheric for a discharge of 30,000 cfs. A point of considerable interest was the action in the river channel downstream. Although the jet leaving the spillway, having an energy content approximating one and a half million horsepower, moved a considerable quantity of loose material in the river channel, it did not attack the banks. (See Figures 4 and 5 of the discussion by Mr. Walter.) The center line of the spillway was virtually on line with the center line of the river so the jet was aimed straight down the river. Were this not the case there would certainly have been bank erosion. The erosion discernible in the foreground of the left bank (Fig. 5, Mr. Walter's discussion) was caused by the jets from one season of operation from the three 96-inch outlet valves which are not aligned with the center line of the river. The results obtained from the above tests are invaluable to the profession;

1. Engineer, Bureau of Public Roads, U. S. Dept. of Commerce, Washington, D. C.

and Mr. Louis Puls who planned and supervised the tests, and Mr. Walter who so kindly presented an account of the operation, are to be highly commended.

The comments of Mr. Blaisdell relating his experiences with small closed conduit spillway structures are most interesting. It is a significant fact that trash racks located in close proximity to a closed conduit spillway usually present a hazard rather than a benefit. As Mr. Blaisdell states, trash collectors should be located well upstream in the lower velocities. Log booms consisting of a single cable stretched across a river, and floated on logs, is the means used on a number of reservoirs of large dams to collect debris. These have proven effective but of course all trash collectors require systematic cleaning.

Although it cannot be stated with absolute certainty, it appears that vortex action observed on models and their prototypes scales up linearly. Mr. Blaisdell's statement that the horizontal tunnel must be submerged for a morning-glory spillway to belch air from the inlet, as demonstrated in Fig. 10, is not corroborated by Mr. Walter who states that the same phenomenon occurred for very small heads on the Hungry Horse Spillway, which was free of submergence. The writer attributes the action to the fact that the spillway acts as a chimney through which a strong draft persists when water is not flowing. Very small water discharges tend to neutralize or reverse the direction of air flow resulting in the accumulation of small pressures in the shaft intermittently permitting reestablishment of the draft, which carries a small amount of spray with it.

The writer's appreciation is extended to Mr. Ballester for calling attention to the spillway at the San Roque Dam in Argentina of which information was lacking. This structure is interesting in that it is designed to operate submerged with as much as 25 feet of head on the crest. Note that the design was verified or developed by model studies, which has been true of practically every morning-glory spillway constructed to date. Mr. Blaisdell and Mr. Ridley will probably be interested in the absence of anti-vortex piers on the San Roque spillway, especially for submerged operation. Mr. Ballester does not enlighten us as to the specific purpose of the 20-inch air pipe which connects to the upper portion of the vertical bend. Is it logical to assume that admitting air at this point materially reduces the intensity of vortex action?

Mr. Ridley's kind cooperation in keeping us well informed on the Manuherikia Falls Spillway in New Zealand is much appreciated. This is apparently the only spillway in which deterioration of the construction joints in the morning-glory and vertical shaft have been reported. Leakage from the reservoir is undoubtedly the principal cause of the difficulty. Mr. Blaisdell's remarks are quite pertinent to the question asked by Mr. Ridley regarding the value of guide piers on the crest of the spillway. For spillways that operate, submerged piers are of definite value. They serve to convert the single large vortex into a number of smaller ones which are less objectionable. Piers, although desired, were not placed on the Owyhee or Hungry Horse Spillway crests because of the complexity introduced by the ring gates. Model tests indicated that piers placed upstream from the ring gates were not sufficiently effective to justify their cost. Piers are of questionable value, however, on spillways operating free of submergence and at relatively low heads.

The writer again wishes to thank the discussors for the interesting and valuable information presented. From the information gained from this

prototype investigation it appears that the point has probably been reached where model studies can now be dispensed with on the more common types of morning-glory spillway design. However, there still remains much to be learned regarding submerged operation, unusual approach conditions, sub-atmospheric crest pressures, and so forth.

Discussion of
"MORNING-GLORY SHAFT SPILLWAYS: PERFORMANCE TESTS ON PROTOTYPE AND MODEL"

by A. J. Peterka
(Proc. Sep. 433)

ALVIN J. PETERKA,[1] M. ASCE—Corrections—In Proceedings-Separate 538 the discussion by F. W. Blaisdell which was contained in the section devoted to the discussion of this paper (433) was actually a discussion of Proceedings-Separate 431 entitled "Morning-Glory Shaft Spillways: Prototype Behavior" by J. N. Bradley. Mr. Blaisdell's discussion of 433 appeared in the section of 538 that was devoted to the discussion of 431.

The comment by Mr. Blaisdell, "Confidence in the use of model studies cannot help but be strengthened as a result of this paper," is of interest because similar tests on a companion structure, Shadehill Dam morning-glory spillway (1) have been made since this paper was written. Of particular interest in these tests (2) was the agreement between model and prototype discharges. A graph, similar to Figure 13 in this paper but containing 12 instead of 4 field measurements, showed remarkable agreement. Average variation between model and prototype discharges was 2.7 percent, with maximum and minimum variations 5.0 and 0.2 percent respectively. These results materially strengthen the conclusion made by the writer that the Heart Butte Spillway model rating curve is accurate.

Other comments by Mr. Blaisdell provide an opportunity to emphasize certain points, which for the sake of brevity, were not fully explained in the paper.

In regard to the filling of the tunnel during the model tests, it should be made clear that the tunnel size had been fixed before the model tests were started. Consequently, no attempts were made to reduce the tunnel diameter during the tests. As Mr. Blaisdell points out, use of a deflector at the base of the shaft, in addition to preventing filling of the horizontal tunnel, is a valuable addition to a structure of this type. The stabilizing effect of the deflector both on partial and maximum flows provides smoothness in operation which under most circumstances would allow reduction in the tunnel diameter with an accompanying reduction in construction cost. The establishment of a positive control point should be the first consideration in the design of any hydraulic structure.

The use of spillway piers is discussed by Mr. Blaisdell, and he states that other antivortex devices are also effective. The writer has been intimately connected with the hydraulic model tests on the Pleasant Hill, Watuaga, South Holston, Heart Butte, Shadehill and Hungry Horse morning-glory spillways and has firsthand knowledge of the tests on Owyhee and Gibson. In each case various other types of antivortex devices were tested but spillway piers proved to be most effective even though in some cases prototype piers were

1. Hydr. Engr., Bureau of Reclamation, U. S. Dept. of the Interior, Denver, Colo.

not constructed. Since many other morning-glory spillways [1] make use of piers in various shapes and forms, it is the writer's opinion that spillway piers, in some form, offer the only known economical and practical solution to the vortex problem. In addition, piers often serve a second purpose as flow guides in unsymmetrical approaches when the morning-glory spillway is discharging freely.

The spray above a morning-glory in operation is, as Mr. Blaisdell points out, usually caused by the outlet being submerged to some degree. Spray may also be seen, however, when the outlet is free. Spray is sometimes caused by a ragged flow surface in the upper portion of the spillway being torn apart by an updraft in the tunnel. At times tunnel spillways act as chimneys, producing an upward draft of air. At low spillway discharges or when the spillway first begins to operate, the air velocity is often sufficient to produce spray above the morning-glory.

The comments by Mr. Blaisdell on the stilling basin side wall flare are entirely correct. No attempt should be made to use rapidly flaring walls such as these without guide walls or other flow spreading devices.

Mr. Blaisdell questions the use of riprap on the prototype channel bottom when the model tests show that serious erosion will not occur. There are two reasons for this: First, the prototype channel consisted of very fine material, having little if any more resistance to erosion than the sand used in the model. With the greater forces in the prototype acting on a much lighter (considering the scale) material, it was feared that deeper erosion might occur in the prototype than was predicted by the model. As insurance against making expensive underwater repairs, riprap was recommended for use on the channel bottom. A second reason was the possibility of degradation of the channel bed. Experience has shown that clear water from a reservoir picks up fine material upon entering the river channel. Degradation of this type has been found to be, in certain cases, as serious as erosion. It is hoped that the riprap will reduce channel degradation close to the structure.

REFERENCES

1. "Morning-Glory Shaft Spillways: Prototype Behavior" by J. N. Bradley, A.M. ASCE, Volume 80, Separate No. 431, April 1954.

2. "Spillway Tests Confirm Model Prototype Conformance" by A. J. Peterka, Engineering Monograph No. 16, October 1954, United States Department of the Interior, Bureau of Reclamation, Denver, Colorado.

Discussion of
"ESTIMATION OF FLOOD PROBABILITIES"

by Leo R. Beard
(Proc. Sep. 438)

LEO R. BEARD,[1] A.M. ASCE—The writer wishes to thank those who have submitted discussions of this paper for their valuable criticisms and for pointing out particulars in which the original paper was not clear or appeared to be erroneous.

Since the time this paper was submitted for publication in 1950, a large amount of work has been done by the Corps of Engineers using methods closely allied to those proposed in this paper. Comments that follow are based in part on findings of these studies.

There are three fundamental questions that have been brought out by the discussers as follows:

(1) There appears to be no a priori reason for using the logarithmic transformation of the normal distribution.

(2) There is a question as to whether the data given in Table 3 relative to the persistence of skew are conclusive.

(3) The question has been raised as to the advisability of using a rigid mathematical method for frequency determinations.

Mr. Ven Te Chow, A.M. ASCE, has discussed the theoretical reasons for using the logarithmic transformation of the normal distribution law in his recent paper entitled "The Log Probability Law and its Engineering Applications."[2] Because so many natural phenomena occur in a logarithmic or percentage relationship to other phenomena and are best thought of in terms of ratios, it is probable that more cases of logarithmic-normal distributions can be found in nature than of arithmetic-normal distributions.

The data contained in Table 3 of the paper were intended to demonstrate that there is no apparent persistence in the skew coefficients of frequency curves constructed for successive periods at a given station. In other words, any indication of skew is accidental rather than characteristic of the location. Because of the lack of satisfactory statistical criteria for measuring the significance of this indication, the data cannot be said to demonstrate conclusively that the skew persistence is zero. However, it is felt that the following conclusions can be drawn:

(1) The estimates based on the log probability method and such short periods of record cannot be improved by studying the skew indicated in the record.

(2) The differences in skew that might actually exist among stations must be very small compared to the quantities indicated in the table.

In regard to the advisability of using a rigid mathematical method, the writer concurs wholeheartedly in the belief that data should not be made to

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2. "The Log Probability Law and its Engineering Applications," Proceedings Separate No. 536, ASCE, Vol. 80, November, 1954.

fit formulas in cases where there is evidence that the formulas do not apply. In any case where such formulas are adopted, the engineer should make certain that available data do not contradict the formula. It is felt that the writer has done this in the basic paper.

The question has been raised as to why the writer did not adopt the Fisher-Tippett distribution of extreme values. There are two reasons why such distribution was not adopted. First, the basic assumptions involved do not hold true when applied to runoff data. It is assumed that each event is the largest of a great number of independent events selected at random. This is not the case in runoff since successive daily flows are almost never independent of each other. Secondly, the theory must be one that can be adapted to runoff volume frequencies, and in the case of long-term volumes, such as 90-day volumes, there are only a few events each year.

Mr. Thom has questioned the validity of the confidence intervals shown in figures 6 and 7. He has apparently misunderstood how they were derived. Errors due to uncertainty of the mean are not added to errors due to the uncertainty of the standard deviations. Rather, their squares are added. This is proper, since the two sources of error are independent of each other. If the specified confidence intervals were in error appreciably, the points shown on figures 4 and 5 would not agree so closely with the curves. It is believed that the method used is more precise than the non-central t method, and is more desirable since required tables are available.

Messrs. Benson and Rantz object to using maximum 24-hour runoff as a frequency variate. The writer, of course, is cognizant of the importance in hydrology of using peak runoff values for the solution of certain problems. It is also important to use long-term runoff volumes in the solution of other problems. A general flood frequency method should be adaptable to both. The only reason that daily values were used in this study is that the data for a great number of long-record stations are readily available. They are surely satisfactory for the purpose. Estimates of peak based on daily observations would not be satisfactory for a fundamental study of this type.

There also was some misunderstanding about the use of plotting positions. It should be emphasized first of all that plotting positions have no bearing on the results of the analysis proposed in this paper. The plotting positions, used for demonstration purposes only, are those based on the same reasoning used in the paper by the writer entitled "Statistical Analysis in Hydrology," ASCE, Trans., Volume 108, 1943.

Nationwide studies, as stated by the writer to be desirable, involve more than has been done by the U. S. Geological Survey. Some comprehensive studies in this direction are being made by the Corps of Engineers. In these studies, not only flood peaks but flood volumes for many durations (up to one year) are being correlated with each other and coordinated over large regions. It is this type of a study that will eventually yield the answer relative to the best frequency methods generally applicable to runoff data.

Discussion of
"STATISTICAL THEORY OF DROUGHTS"

by E. J. Gumbel
(Proc. Sep. 439)

E. J. GUMBEL[1]—The contributions of Messrs. Perry M. Ford, R. D. Goodrich and Shuh-Chai Lee are quite pertinent. Mr. Ford is right: the severest drought need not happen in the driest year, just as the highest flood may happen in a relatively dry year. This disadvantage of the method used is due to the definitions. To overcome it the minimum discharges for say ten consecutive days could be analyzed by the method used in the article.

In addition, Mr. Ford raises a two-dimensional problem, the joint distribution of severity and duration, which are probably interconnected. As far as the writer knows, this problem has not yet been solved, neither for the floods or for the droughts.

Mr. Ford is also right in stating that a priori any one of the 365 days of the year has the same chance to produce the flood or the drought. This is not contradicted by the a posteriori existence of seasonal variations, which limit the analysis of the droughts to the dry season.

His question about the meaning of the word asymptote can easily be answered: in geometry an asymptote is a curve (in particular a straight line) which is approached by another curve. For example, the hyperbola approaches two straight lines. In the same graphical sense, a statistical distribution curve, such as the normal one, approaches the abcissa for large absolute values of the continuous variate if the variate is unlimited, while it becomes identical to the abscissa if the variate is limited. The difference between the two cases is hardly visible.

In calculus a certain function or a certain value reached under a limiting condition is called asymptotic. In the same way, a statistical distribution may possess an asymptotic expression if a certain parameter or if the variate increases without limit. The distribution used in the article is the asymptotic distribution of the smallest value of a limited variate. Here a parameter, namely the number of observations, increases without limit. Asymptotic expressions may thus exist for limited and unlimited variates.

Mr. R. D. Goodrich recalls his formula of 1927, which may be written in the designations used in the article

$$1 - F(x) = \exp\left[- k(x - \epsilon)^{\alpha} \ (\omega - x)^{-\beta} \right]$$

It contains 5 parameters k, ϵ, ω, α, β, two of which have a clear meaning, since ϵ is the lower limit and ω is the upper limit of the variate. It would be difficult to find a statistical series which would resist such a powerful formula, although the fitting of five parameters induces very large errors of estimation. The derivation of this formula is purely empirical. Now, it is

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well known that any series of observations can be reproduced by such formulae or development by series. However, within this method, nature takes its revenge for the lack of man's thinking. The closer the approximation within the observed range, the freer becomes the formula outside of this range, and extrapolation even within a very short distance becomes impossible. Now, a formula is senseless if it cannot be used for a forecast, and this marks the difference between an interpolation formula and a statistical law, although both have the form of a mathematical equation. A statistical law is based on the fundamental nature of the variate, and a large part of the progress of mathematical statistics in recent years consists in the elimination of formulae which have no logical basis.

No such foundation exists for the Goodrich scheme—which does not mean that it cannot work. Just the opposite, it must work sometimes, since for $\beta = 0$ Goodrich's formula is but another form of equation (4.1) and for $\beta = 0$ and $\epsilon = 0$ the special case (2.1) is reached. Therefore, the analogies stated by Mr. Goodrich exist, and he may even claim to have used the theory of extreme values—without realizing it.

The writer wishes and hopes that Mr. Shuh-Chai Lee is right in using the t test instead of the control curves which the writer proposed for the floods. Unfortunately, the writer was unable to prove that the t distribution constructed for normal values holds also for the extremes. Therefore, he refrained from applying this test. If the statement can be proven, it will mean a considerable simplification in the use of control curves.

Mr. Lee points out that the calculation of the mean and the standard deviation of the logarithms permits plotting the droughts of all rivers as one straight line, provided that the lower limit is zero, and asks whether this simplification could be extended to the case of a non-vanishing limiting value. Such a procedure is possible but hardly advisable. The logarithms of the differences between the droughts and the minimum as functions of y should be scattered about straight lines, which could be standardized. However, since the lower limit must be estimated, the amount of work involved in this procedure is exactly the same as in the usual one, and the advantage of the uniformity is lost by the differences in the lower limits for different rivers.

Professor John J. Gannon, School of Public Health, University of Michigan, Ann Arbor, Michigan, drew the writer's attention to a misprint. The variances of the droughts for the Connecticut River given in table 5, third line, is 110559.5673 and not 1105595.673. The writer hopes that not too many readers have been confused, since the standard deviation 332.5 is correctly given in the next line.

After finishing the paper, the following difficulty occurred: The estimation (4.8) of the lower limit does not insure that ϵ is smaller than the smallest observed drought x_1. If x_1 turns out to be smaller than ϵ, the estimate (4.8) should be replaced by the following unbiased estimate

$$(4.10) \qquad \hat{\epsilon}_1 = (x_1 - C\hat{u})/(1 - C); \quad C = \left(\log \frac{n}{n-1}\right)^{1/\hat{\alpha}}$$

which is obtained from the plotting position $1 - 1/(n + 1)$ of the smallest drought and where $\hat{\alpha}$ is the estimate obtained from (4.6'). However, the use of (4.10) requires a new estimate of α. This is obtained by introducing $\hat{\epsilon}$ in (4.8), which may be written

$$(4.11) \qquad B(\hat{\alpha}_1) = (\hat{u} - \hat{\epsilon}_1)/s$$

where s is the sample standard deviation. Table IV leads to an estimate $\hat{\alpha}_1$, which is again introduced in (4.10) and leads to an estimate $\hat{\epsilon}_2$, etc. The procedure converges quickly and insures that the minimum is smaller than the smallest observation, as it should be.

Since the discussion did not raise objections against the theory, the writer hopes that the procedure advocated will be useful for the engineers and that these methods will be applied in the future.

Discussion of
"THE LOG PROBABILITY LAW AND ITS ENGINEERING APPLICATIONS"

by Ven Te Chow
(Proc. Sep. 536)

LEO R. BEARD,[1] A.M. ASCE, and H. A. KEITH[2]—This paper is obviously the result of a great deal of research and thought on this subject by the author and should prove to be of permanent value in the field of engineering statistics. The following comments are based on some experience by the writers in the application of the Log-Probability Law to hydrologic data.

The author has stated the theory advanced by Laplace that the distribution of the sum of a large number of independent variables approaches the "normal distribution" regardless of the distributions of the individual variables. This law explains why the normal law of error is represented by so many natural phenomena. The author goes on to demonstrate how, by the same reasoning, the product of a large number of variables would be distributed in accordance with the logarithmic normal law of error, regardless of the distributions of the individual variables. He recognizes also that there are many natural phenomena that are neither logarithmically nor arithmetically normally distributed and has proposed a method for fitting data for any of these distributions by simply expressing the logarithmic normal distribution in terms of the coefficient of variation of the arithmetic values for a given coefficient of skew of the arithmetic values. He suggests that this resulting function be used for any coefficient of variation, although the logarithmic normal distribution would correspond to only one particular coefficient of variation (for each coefficient of skew). This procedure, although arbitrary, has the advantage of providing a continuously varying (and consequently mutually consistent) series of curves for fitting any distribution between the normal and log-normal, and beyond the log-normal. It also has the advantage, as the author points out, of being capable of fitting distributions having any combination of mean, coefficient of variation, and coefficient of skew. The disadvantage of lacking a complete theoretical justification is perhaps of secondary significance at the present time, in view of the fact that foremost statisticians appear to disagree on the virtue of such theoretical considerations.

The extreme-value distribution developed by Tippett and Fisher is dependent more on theoretical considerations, in that the skew coefficient is fixed by those considerations. It has been applied extensively in hydrology, especially in connection with the distribution of annual maximum discharges of a stream. The basic assumptions of the theory are not strictly applicable in such cases, since the hydrologic events on a stream within a year are not entirely independent of each other. This becomes of more concern as annual maximum runoff volumes for the longer durations are analyzed (since there are fewer events per year). Each annual maximum event cannot be assumed

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to be the largest of a large number of independent events, which is the basic assumption of the extreme-value theory. This break-down in the application of the extreme-value law is very serious from a hydrologic standpoint, and a more comprehensive method such as that proposed by Mr. Chow is required.

A very serious limitation in the method proposed by the author and of methods based on the extreme value law is that fitting markedly skew distributions by the method of moments is highly unreliable. (See R. A. Fisher, Statistical Methods for Research Workers). For this reason, the writers have preferred fitting distributions that approximate the log-normal distribution by use of moments of the logarithms. Hydrologists have long felt the inherent danger in using means, coefficients of variation, and skew coefficients based on annual maximum flows, because the inclusion or exclusion of one large event would drastically change the results. This effect is not so serious when the coefficients are based on the logarithms, since they are approximately normally distributed.

In this connection, it is interesting to note that the author's suggestion that the computed skew coefficient be adjusted graphically by plotting on log-probability paper is reasonable only because the skew coefficient is more reliable when referred to the logarithmic scale (as long as the distribution is approximately log-normal). If the coefficient were based on the logarithms, such graphical adjustment could be of no help. It is not clear what advantage is to be gained by replotting the curve on a special grid so designed that the curve will plot as a straight line, since the curve has already been accurately determined during the process of adjusting the skew coefficient.

If the moments of logarithms are used, then the tables of K values developed by Mr. Chow would not be applicable theoretically, unless some transformation of the coefficients is used. From a practical standpoint, however, they can be used and may prove to be entirely satisfactory. The writers have had some success in using the Pearson Type III distribution (which is very similar) for cases where the distribution appears to be slightly skewed. A theoretical discussion of the Pearson Type III distribution and its relation to the normal distribution (which is a special case of the Pearson Type III distribution) is given by W. Palin Elderton in his book, Frequency Curves and Correlation, (Cambridge U. Press, 1938).

The coefficient of variation was developed for the purpose of comparing the variability of stream flows at one location with the variability at another location. Because of the large difference in flow magnitudes among the various rivers, it was necessary to make this comparison on a percentage basis; hence the coefficient of variation. It should be noted that the standard deviation of the logarithms provides a measure of variability that is independent of the general magnitude of flows, and is therefore comparable in this respect to the coefficient of variation. It should be superior to the coefficient of variation when used in connection with distributions approximating the log-normal, in that it is a more reliable statistic.

The index of variability developed by Lane and Lei, has been shown by the author to be theoretically equivalent to the standard deviation. However, it probably is not as efficient a statistic as the standard deviation when used in connection with the normal distribution, and appears to be considerably more work to obtain. The writers prefer the procedure of calculating the standard deviation from the observations directly (or from the logarithms) by use of standard short-cut methods (in which it is not necessary to calculate each individual deviation from the mean).

E. J. GUMBEL[1]—This paper is very valuable because it draws the attention of the engineer to an important distribution and shows practical ways to use it. The larger part of its exposition is clear and the long list of references very helpful. However, certain wordings are unclear and liable to misunderstanding. For example, the author claims that the standard deviations are additive for independent variates, while this holds for their squares, the variances. The equality between the probabilities of the normal and the log normal variate is presented as a result, although it is the basis for the transformation and leads to the transformed distributions. In the formulae for the moments a factor M_0 is used, which can be omitted, since its value is by definition equal to unity.

For the estimation of the skewness, the author gives Hazen's correction but advocates a multiplicative factor $n/(n-1)$, while the correction used in modern textbooks of mathematical statistics, $n^2/(n-1)(n-2)$, is not given.

It is difficult to see why certain methods which have been replaced by better ones are mentioned, such as the plotting position $(2m-1)/2n$. The plotting position $m/(n+1)$ the use of which "the author suggests" was derived previously and discussed in many publications, e.g., Weibull in 1949 and used in the "Review of Flood Frequencies Methods," Paper 2574, Trans. Amer. Soc. of Civil Eng., 1951.

The foregoing remarks are not meant to diminish the practical importance and the merit of this contribution for the Engineering profession.

J. F. McILLWRAITH[2]—The writer desires to express his appreciation of the author's theoretical treatment of the log-probability law and its application to the analysis of engineering data.

Following the writer's paper on New South Wales rainfall intensities,(1) a study was recently carried out of the daily rainfalls from a number of Australian stations. In this investigation the first basic assumption tested was:

> "The logarithms of the primary rainfalls are normally distributed for fixed location and duration"

The daily primary rainfalls from 141 stations at least 20 miles apart and each having 60 or more years of record were analysed. This gave a total of 9915 station years of record.

The log-probability law was adopted for this investigation following many trials with various transformations of the variate.

Logarithmic transformation for the analysis of rainfall data appears to be justified, as the following tests will indicate.

Index of Skewness. Unbiased estimates of the ratio

$$z_1 = \frac{g_1}{S.E._{g_1}}$$

where g_1 is the index of skewness and $S.E._{g_1}$ is the standard error of g_1, were calculated for each of the 141 stations. Fig. 1 shows graphically the distribution of z_1 with a normal curve fitted to the data and for comparison a normal curve of mean zero and standard deviation of 1.00 is also shown. The mean of +0.745 and the standard deviation of 1.10 show a highly significant difference from the mean of 0.0 and standard deviation of 1.0 as expected

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from a normal distribution.

The distribution of g_1 for samples from a normal population is a symmetrical leptokurtic curve which tends to the normal fairly rapidly as n increases. Since the period of record for all stations is in excess of 60 years it would be reasonable to expect the distribution of z_1 to approximate a normal curve.

Pearson[2] has given a table for the approximate 5% and 1% probability points for g_1. Based on this table of Pearson's, table 1 lists the number of stations for which g_1 is significantly high.

TABLE 1

Number of Stations for which g_1 is significantly high.

Number of stations with significant positive skewness			Number of stations with significant negative skewness		
	5%	1%		5%	1%
Observed	28	8	Observed	2	0
Expected	7.05	1.41	Expected	7.05	1.41

Table 1 supports the contention that the distribution of the logs. of the primary rainfalls shows an overall tendency towards positive skewness.

Index of Kurtosis. Unbiased estimates of the ratio

$$z_2 = \frac{g_2}{S.E.g_2}$$

where g_2 is the index of kurtosis, $S.E.g_2$ is the standard error of g_2 were calculated for each of the 141 stations. Fig. 1 shows graphically the distribution of z_2 with a normal curve fitted to the data and for comparison a normal curve of mean zero and standard deviation of 1.0 is also shown.

The distribution of g_2 in samples from a normal population is an extremely skew curve even when the samples are relatively large. In the absence of satisfactory tables for the confidence limits of g_2 for samples less than n = 100, it would appear that the distribution under review does not abnormally depart from expectation.

Ratio of mean deviation to standard deviation. Geary[3,4] has shown that for small samples a good test for normality is the ratio

$$z_3 = \frac{M.D.}{S}$$

where M.D. is the mean deviation of the sample, S is the standard deviation of the sample. He has given a table[4] showing the 10%, 5% and 1% probability points for this ratio. Based on this table of Geary's table 2 lists the number of stations for which this ratio is significant. This distribution shows a preponderance of stations for which this ratio is significantly low.

TABLE 2

Number of stations for which ratio z_3 is significant.

Number of stations for which ratio z_3 is significant in upper limits.				Number of stations for which ratio z_3 is significant in lower limits.		
	10%	5%	1%	10%	5%	1%
Observed	5	5	2	34	18	4
Expected	14.1	7.05	1.41	14.1	7.05	1.41

Chi-squared test. A normal curve for the logs. of the primary daily rainfalls with mean and standard deviation estimated from each individual station was fitted to the records from each of the 141 stations. The area under this normal curve was then subdivided into 6 equal areas such that the number of heavy primary rainfalls in each section of the curve would be $\frac{N}{6}$ where N is the number of years of record for each individual station. A chi-square test was then applied as a measure of the goodness of fit between actual and anticipated number of storms in the six classes. In this test, neglecting possible correlation between adjoining stations, the number of degrees of freedom was taken as 3, i.e. 3 less than the number of classes. Values of χ^2 were calculated for each station and the corresponding probability, P, for χ^2 to be exceeded evaluated.

The distribution of P for χ^2 is listed in table 3. In the calculation of χ^2 no provision was made for Yates' correction for continuity.

TABLE 3

Distribution of P for χ^2 calculated for goodness of fit of the distribution of primary rainfalls from 141 stations. $\nu = 3$

	Range of P									
	0.0 to 0.1	0.1 to 0.2	0.2 to 0.3	0.3 to 0.4	0.4 to 0.5	0.5 to 0.6	0.6 to 0.7	0.7 to 0.8	0.8 to 0.9	0.9 to 1.0
Anticipated number of stations	14.1	14.1	14.1	14.1	14.1	14.1	14.1	14.1	14.1	14.1
Actual number of stations	24	12	21	17	18	12	12	15	7	3

This distribution is also shown graphically in Fig. 1. The value of χ^2 in table 3 combining the last two columns is 24.7 for which the probability for χ^2 to be exceeded with 8 d.f. is 0.003, which figure is very low. The sample

of 9915 station years is, however, very large and the result is not as bad as at first sight appears. Observational and sampling errors, which are always abnormally high in rainfall observations, might reasonably account for some of the stations having a low value of P whilst a value approaching 1.0 is impossible if Yates' correction for continuity is ignored, unless the number of years is a multiple of 6. For instance if the period of record is 63 years the expected number of observations in each class would be $\frac{63}{6} = 10.5$, whereas in a perfectly normal distribution the closest approach to the number would be 10 or 11.

The 3 d.f. assumed from the data for each station may not represent that much independent information. For instance the standard deviations and probably the means for stations only 20 miles apart may not be independent. This contention was tested by drawing a graphical distribution for the individual chi-square values for the 141 stations. For comparison the true chi-square distributions for 3, 4 and 5 d.f. were also charted. This graph indicated that 4 d.f. would be more appropriate and table 4 gives a comparison of the grouped chi-square actual values with the expected values using 4 d.f.

TABLE 4

Comparison of actual grouping of X^2 with expected grouping with 4 d.f.

	Range of χ^2					
	0.0 to 1.5	1.5 to 3.0	3.0 to 4.5	4.5 to 6.0	6.0 to 7.5	>7.5
Anticipated number of stations 4 d.f.	24.1	38.2	31.4	20.4	11.6	15.3
Actual number of stations.	27	44	31	13	11	15

The value of X^2 for table 4 is 3.95 for which the probability for X^2 to be exceeded is 0.55 with 5 d.f. This justifies the use of 4 d.f. instead of 3 in the chi-square test summarized in table 3.

Adopting a value of 4 for the degrees of freedom, the bottom row in table 3 would then be amended to read 15, 11, 11, 9, 16, 20, 15, 11, 21 and 12 for which the value of X^2 is 10.32 and P = 0.32 with 9 d.f.

Table 4 and the graph from which it was derived support the findings of H. Chernoff and E. L. Lehmann[7] whose investigations indicate in the test under review that:

(i) the approximate number of degrees of freedom on the simple theory is somewhat larger than anticipated;
(ii) nominal significance levels are altered, i.e. the tests are less stringent than one thought.

Kolmogorov's Statistic. Kolmogorov's z statistic[5,6] tests the cumulative distribution of the variable. Applying this test to table 3 with the bottom row

adjusted to the numbers given above for 4 d.f. a value of 0.868 is obtained for z. The value of the z distribution for $z = 0.868$ is $L(z) = 0.564$. Thus the probability of obtaining $z = 0.868$ in a sample observation is approximately 0.44.

Comparison with observed heavy rainfalls. The probability that the logarithm of a heavy daily rainfall is equal to or greater than the mean of the logarithms of the daily primary rainfalls plus 3 standard deviations based upon a normal distribution is approximately 0.00135. This probability may be written in conventional symbols

$$P_r \left\{ \log R \geq (\overline{\log R} + 3S) \;\middle|\; (N.D.) \right\} \doteq 0.00135$$

where R is the daily primary rainfall in points, and S is the Standard deviation of the logs. of the primary rainfalls. A probability of 0.00135 corresponds to a recurrence interval of 742 years so that in 9915 station years of record one would expect to find 13.4 primary rainfalls having a departure from the mean equal to or greater than 3 standard deviations after logarithmic transformation. Actually there were 18 of which 4 occurred from the same storm in 1910 at stations within a radius of 40 miles. These four heavy falls would therefore be highly correlated.

Similarly

$$P_r \left\{ \log R \geq (\overline{\log R} + 2S) \;\middle|\; (N.D.) \right\} \doteq 0.0228$$

A probability of 0.0228 corresponds to a recurrence interval of 43.9 years so that in 9915 station years of record one would expect to find 226 primary rainfalls having a departure from the mean equal to or greater than 2 standard deviations. Actually there were 273, which is not unreasonable having regard to correlation between stations more than 20 miles apart.

CONCLUSIONS

It is considered the distribution of the logs. of the primary rainfalls is sufficiently close to normal to justify the logarithmico-normal distribution for all engineering investigations associated with rainfall intensity-frequency studies.

The writer is indebted to the author for his mathematical interpretation of the law which is applicable to many hydrological studies. In the case of most other hydrological data, such as flood flows, the distribution is more skew than in the case of primary rainfalls.

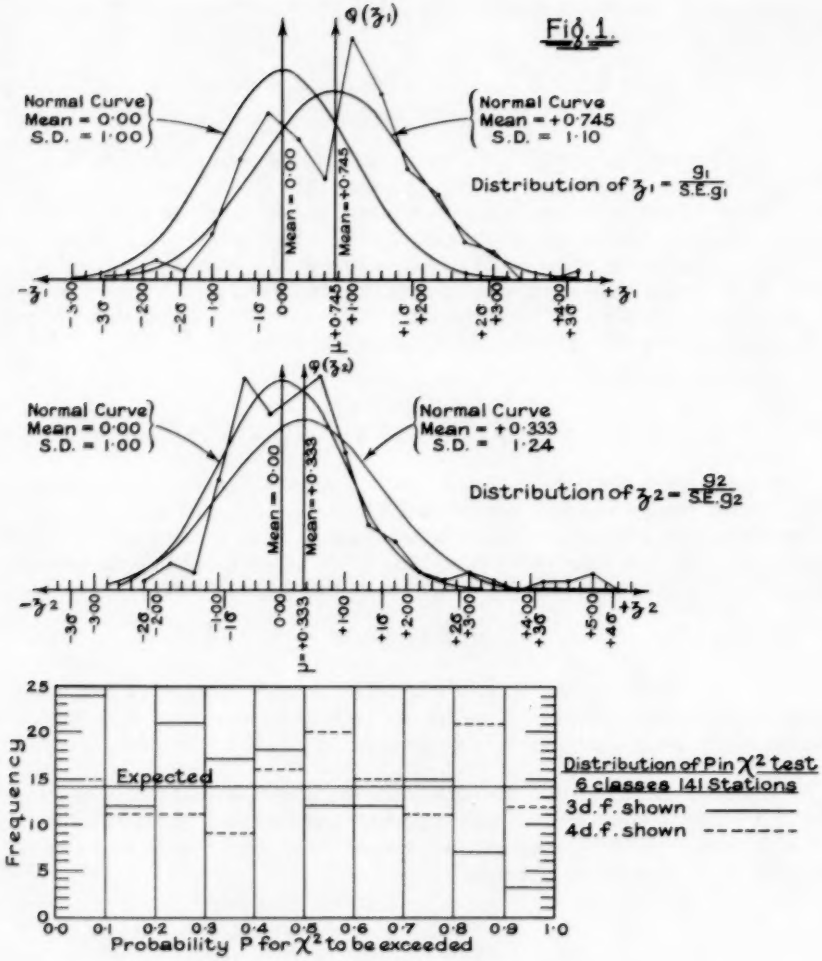
REFERENCES

1. "Rainfall Intensity-Frequency Data for New South Wales Stations," J. F. McIlwraith, Jour. Inst. Engrs. Aust. July-Aug. 1953, pp. 133-139.

2. "A Further Development of Tests for Normality," E. S. Pearson, Biometrika V. 22 (1930-31), pp. 239-249.

3. "The Ratio of the Mean Deviation to the Standard Deviation as a Test for Normality," R. C. Geary, Biometrika Vol. 27 (1935), pp. 310-332.

Fig. 1.


Normal Curve
Mean = 0·00
S.D. = 1·00
Normal Curve
Mean = +0·745
S.D. = 1·10
Distribution of $z_{g_1} = \frac{g_1}{S.E.g_1}$
Mean = 0·00
Mean = +0·745
Normal Curve
Mean = 0·00
S.D. = 1·00
Normal Curve
Mean = +0·333
S.D. = 1·24
Distribution of $z_{g_2} = \frac{g_2}{S.E.g_2}$
Mean = 0·00
Mean = +0·333
Frequency
Expected
Probability P for χ^2 to be exceeded
Distribution of P in χ^2 test
6 classes 141 Stations
3d.f. shown
4d.f. shown

4. "Moments of the Ratio of the Mean Deviation to the Standard Deviation for Normal Samples," R. C. Geary, Biometrika Vol. 28 (1936), pp. 295-307.

5. "On the Kolmogorov-Smirnov Limit Theorems for Empirical Distributions," W. Feller, A.M.S. Vol. 19 (1948), pp. 177-189.

6. "Table for Estimating the Goodness of Fit of Empirical Distributions," N. Smirnov, A.M.S. Vol. 19 (1948), pp. 279-281.

7. "The Use of Maximum Likelihood Estimates in X^2 Tests for Goodness of Fit," H. Chernoff and E. L. Lehmann, A.M.S. Vol. 25 (1954), pp. 579-586.

F. A. BERTLE,[1] J.M. ASCE—The thorough mathematical analysis relating statistical parameters computed from arithmetic units to the characteristics of a logarithmic-normal distribution and the copious list of references in Mr. Chow's paper will be of value to anyone preparing hydrologic frequency studies.

A typographical error appears in equation (38), which is the equation used by Hazen for C_v^2 instead of C_v as shown. Equation (39) should include the adjustment factor, equation (43), to agree with Hazen's usage because the values of C_s in Hazen's table of frequency factors are values of adjusted C_s.

To evaluate the effect of the statement made in step (2) on page 11, the following two comparisons were made:

(1) Figure 4 shows the logarithmic-normal distributions shown in the two tables for several values of C_v. Hazen's lines are steeper than Chow's with the difference increasing as the C_v becomes larger. It was noted here that even though the Chow line has a flatter slope, Chow's method of plotting points for a given sample will make the array of points assume a steeper slope than will Hazen's method of plotting the same points. If samples containing 25 points that plot on these lines are selected, the statistics computed from the samples will not agree with the labels given by Chow to his lines, but will agree closely with the labels given by Hazen to his lines. A summary of one of these comparisons is shown in Table 5.

(2) Utilizing data in Bureau of Reclamation files, the writer plotted the computed curves, as indicated by the two tables, on the same sheet with the array of points representing observed annual-peak stream flows at several stream gages. In most cases, the Hazen curves fit the array of points better, particularly at the upper end. One example is shown on Figure 5.

Comparison of the Hazen and Chow tables shows that the differences are probably due to the number of items in the sample used to compute the statistics. Pairs of parallel lines on logarithmic-probability paper can be selected from the two tables to represent the same logarithmic-normal distribution. Chow's line will be slightly lower than Hazen's line and will be labeled with larger values of C_v and C_s. Increasing the number of items in samples from the same logarithmic-normal population increases the arithmetic mean, C_v, and C_s as computed from the sample because there is an increased probability of including large and small extremes in the larger samples. In a logarithmic-normal distribution, the added extreme logarithmic values are equidistant above and below the mean logarithm. However, when these logarithm departures are converted into arithmetic units, the points above the mean have larger arithmetic departures than those points below the mean. An example to illustrate this effect is shown in Figure 6. The basic line represents a logarithmic-normal distribution with a standard

1. Engr., Bureau of Reclamation, U.S. Dept. of the Interior, Denver, Colo.

deviation of natural logarithms equal to 0.92. By using equations (14) and (15) this line has a C_v of 1.155 and a C_s of 5.0. Using the frequency factors shown in Chow's table, this line was replotted as shown by the lower dashed line. It will be noted that this line is lower than the basic line only because the scale of ratio-to-mean is referred to the arithmetic mean instead of to the mean of the logarithms. By using Hazen's equation for plotting points and tables of the probability integral, samples were selected from the basic line having 5, 25, and 100 items. The values of each point in each sample were converted to arithmetic units, the statistics were computed, and the lines were plotted as shown. As the number of items in the sample increases, the lines approach Chow's line, and the computed values of C_v and C_s approach Chow's label for his line. While the statements and example just presented represent the average expected results, it should be understood that actual samples will often depart considerably from the statistical model. The example does, however, illustrate the need for adjustment factors to convert sample statistics into population parameters. Hazen's adjustment factor and his table of frequency factors were based on 25-point samples and have given remarkably good results in most studies of streamflow where the length of record is short (20 to 40 years). Chow's table is the exact representation of the population parameters, but, to be used for small samples, it requires the establishment of the proper correction coefficients to be applied to the sample statistics.

This paper recommends using plotting points based on equation (42) instead of that used by Hazen, equation (40). The selection of plotting points is a procedure for choosing varying class intervals for grouping the data such that one event can be expected to occur in each class interval. The Hazen equation, which is also recommended by Hald,[2] assumes that each event can be plotted near the center of the class interval. Sample statistics were computed from pairs of 25-point samples which had been selected by use of the two equations from lines on probability paper such as those shown in Figure 4. Results of three of these comparisons are shown in Table 5. As can be seen from this table, the C_v and C_s values computed from the sample selected by Hazen's equation for plotting points agree closely with the population C_v and C_s shown in Hazen's table of frequency factors. The sample selected by Chow's equation for plotting points gives smaller values of C_v and C_s.

The method described on page 12 proposes the construction of a special probability scale after the graphic C_s has been determined, so that a straight line can be used to represent the frequency distribution. Since the graphic C_s has already been selected, the curve can be drawn with sufficient accuracy from the table of frequency factors to give the value at any selected frequency. Construction of a special probability scale and the resulting straight line will not change these results and consequently appears to be unnecessary.

Since the examples cited above show that, for small-size samples, Hazen's values are consistent and fit the observed records as well as, if not better than, Chow's proposal, it is concluded that this paper does not justify discarding Hazen's table. Rather, it appears to be a challenge for the development of the factors needed to adapt Chow's table to the statistics computed from small samples.

2. "Statistical Theory with Engineering Applications," by A. Hald, John Wiley & Sons, Inc., New York, N. Y., 1952, p. 131.

Table 5--Comparison Between Population and Sample Statistics[1]

Type of Distribution	Population Parameters[4]				Statistics from 25-point Sample			
	From Table 1 (Hazen)		From Table 2 (Chow)		Points Selected by Equation (40)[2] (Hazen)		Points Selected by Equation (42)[3] (Chow)	
	C_v	C_s	C_v	C_s	C_v	C_s	C_v	C_s
Arithmetic -normal	1.0	0	1.0	0	0.995	0	0.906	0
Logarithmic -normal[5]	0.98	2.7			0.99	2.69	0.86	2.12
			1.078	4.5	0.95	2.61	0.83	2.04

1. Formulas:

$$C_v = \frac{1}{\bar{X}} \sqrt{\frac{\Sigma \Delta^2}{N-1}}$$

$$C_s = \frac{\left(1 + \frac{8.5}{N}\right) \Sigma \Delta^3}{(N-1)\, C_v^{\,3}}$$

2. $P = \frac{2M - 1}{2N} 100$

3. $P = \frac{M}{N+1} 100$

4. Populations were assumed to be represented by straight lines on arithmetic--or logarithmic--probability paper as plotted from frequency factors for given C_s value as shown in either Hazen's or Chow's table.

5. These two lines were chosen to intersect at 50 percent probability (have the same mean logarithm). Line from Hazen's table is steeper than line from Chow's table.

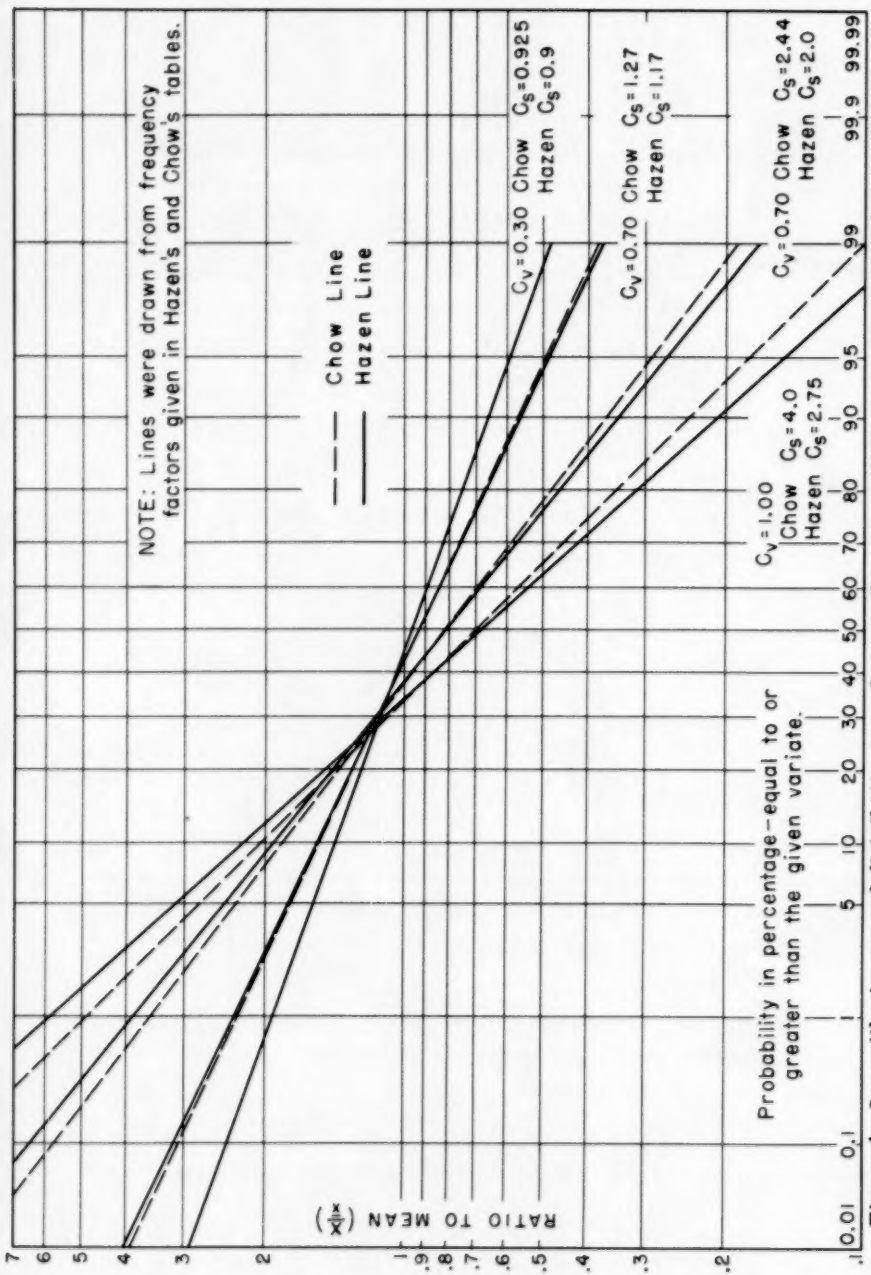


Figure 4. Logarithmic-normal distributions as shown in Hazen's and Chow's tables of frequency factors.

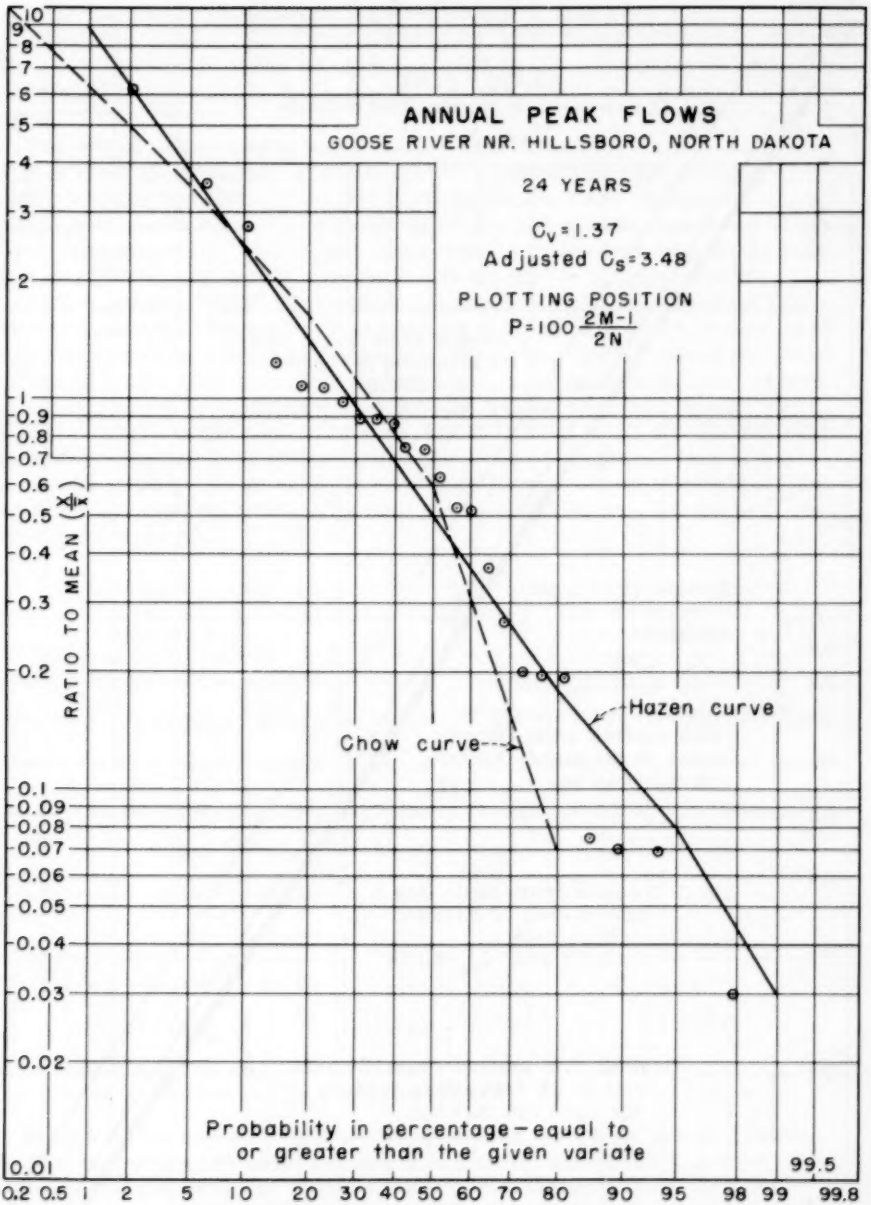


Figure 5. Computed frequency curves of annual peak stream-flows for Goose River near Hillsboro, North Dakota.

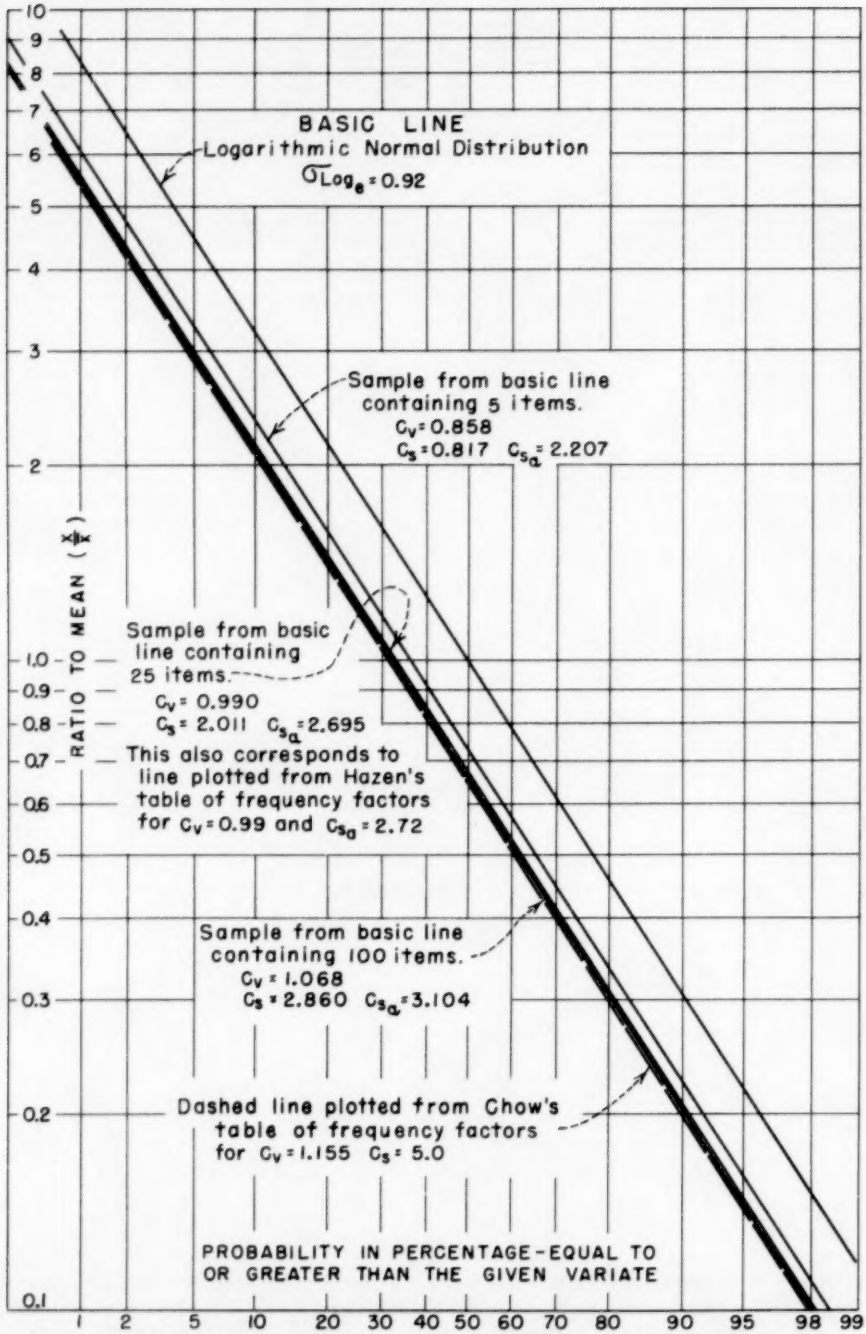


Figure 6. Effect of sample size on computed statistics for a logarithmic-normal distribution.

RALPH W. POWELL,[1] M. ASCE—The mathematics of this paper is beyond the ability of most of us to check, but unless the experts find errors, we will assume that the author's development is correct. It is very interesting that the frequency factors that Hazen asked for in 1921, are now supplied in 1954. They are enough like those which Hazen worked out empirically to give us confidence in the method, but enough different to make the revision worthwhile.

The list of references is as the author says "generous," but like all such lists it could be extended. For instance it is interesting that Howland (1) in 1943 noted the fact that curves like those in Figure 2 are straight for C_v = 0.35, bend upward when C_v is more than 0.35, and slightly downward when C_v is less than 0.35. This figure was evidently discovered without the help of mathematics, and agrees very well with the author's figure of 0.364.

Also it would seem that the author's formula for plotting positions was first proposed by Kimball.(2) It is perhaps not generally realized how many different positions have been suggested. The following table gives the probability to be assigned to each of the ten yearly maxima of a ten year record as recommended by various authorities. (It will be generally agreed that a ten year record is too small a sample, but is used to illustrate the different methods.) The left hand column gives the order of magnitude (say of the maximum stream flow in each year) arranged from largest to smallest, and the other columns the probability that this flow will be reached the next (or any other) year.

	Horton Recurrence	Gumbel	Kimball-Chow	Beard	Hazen	Horton Exceedence
1	0.100	0.100	0.091	0.067	0.050	0.000
2	0.200	0.189	0.182	0.164	0.150	0.100
3	0.300	0.278	0.273	0.258	0.250	0.200
4	0.400	0.367	0.364	0.355	0.350	0.300
5	0.500	0.456	0.455	0.452	0.450	0.400
6	0.600	0.544	0.545	0.548	0.550	0.500
7	0.700	0.633	0.636	0.645	0.650	0.600
8	0.800	0.722	0.727	0.742	0.750	0.700
9	0.900	0.811	0.818	0.836	0.850	0.800
10	1.000	0.900	0.909	0.933	0.950	0.900

In view of the uncertainty of the whole matter the writer makes bold to suggest that engineers adopt the policy of plotting annual flood records on probability paper (whether Hazen's logarithmic, Goodrich's,(3) or the writer's) not as a point, but as a line extending from the recurrence probability to the exceedence probability, and thus including all the plotting positions that have been suggested. The highest line would go off the paper at the right, and the lowest off the paper to the left. The writer believes that a line

1. Prof. of Eng. Mech., The Ohio State Univ., Columbus, Ohio.

or curve drawn through these lines by an experienced engineer will be about as close to the true probability as we can expect from our relatively short sample. He certainly doubts that engineers will go to the trouble of constructing a special probability paper for each problem as suggested by the author.

REFERENCES

1. "Methods of Estimating Flood Frequencies" by W. E. Howland, Civil Engineering, April 1943 (Vol. 13), p. 185.
2. "Assignment of Frequencies to a Completely Ordered Set of Sample Data" by Bradford F. Kimball, Trans. Am. Geophysical Union, December 1946 (Vol. 27), p. 844.
3. "Straight Line Plotting of Skew Frequency Data" by R. D. Goodrich, Trans. ASCE, Vol. 91 (1927), p. 1.

WILLIAM H. SAMMONS,[1] J.M. ASCE—
We are indebted to Chow for his competent demonstration of useful tables, methods, etc., designed to encourage the use of the logarithmic-probability law. Although the arithmetic mean is simple to compute, the computation of the other parameters is another matter. The writer finds even the so-called "short-cut" methods tedious and many other hydrologists feel the same.

The writer prefers methods based on the theory of least squares, or some variation thereof. With graphical methods, it is difficult to select a solution which is void of personal judgment. Unconsciously, it is possible to become biased in fitting the theoretical curve to the observed curve. Some points are not given their proper weight.

A recent paper, (1) "Estimation of Flood Probabilities" by Leo R. Beard, A.M., ASCE, includes many points of interest not covered by the author in his paper. Discussion by Messrs. H. C. S. Thom, M. A. Benson, and S. E. Rantz, and G. N. Alexander in Proceedings Separate No. 568, December, 1954, brings out additional information which should be worthy of consideration by Chow in the closing discussion of his paper. The use of the "non-central" t by Thom is a very good point that should be considered in control of the newly proposed method of the author. A curve without confidence bands is of more limited value than one with selected confidence bands. The extreme value theory of Gumbel and Chow lends itself readily to the application of confidence bands to the theoretical curve.

The use of the author's equation

$$x = \bar{x} + \sigma_x K \qquad \text{.....(28)}$$

will adequately solve most problems provided that a K-T curve is plotted for the data under investigation. Long periods of records are of prime importance when dealing with any method of analysis. If twenty or more items are not contained in the sample, the variation in slope of a theoretical curve can vary so much that only periods of record for the same period of time have any relationship with each other. Studies dealing with the development of

1. Research Engr., Materials Research Lab., Commonwealth of Kentucky, Dept. of Highways, Lexington, Ky.

synthetic data[2,3] from 24-hour non-recording rainfall stations have verified that only records for the same period of time give acceptable correlation and low standard errors of estimate.

Equation (28) and K-T curves result in the best correlation between durations. Since the K-values were determined graphically, the standard error of estimate is usually higher than would be obtained from data based on the theory of least squares. Individual station records, when analyzed by the K-T curve method of the authors, usually lead to a theoretical curve which is not linear. This curve may be concave upward or downward when plotted on extreme probability paper, thus indicating that the data might very well be analyzed (linearized) by other methods. These methods could include the arithmetic normal (An) or the logarithmic normal (Ln) type distributions.

In the hydraulic and hydrological fields of study by the Division of Research, Kentucky Department of Highways, over 660 sets of data have been investigated in the past two years.

Three-hundred fourteen (314) sets of data were analyzed for various periods of record for 24-hour rainfall maxima from non-recording rainfall stations located in ten states. Twenty-four stations, with recording gauges, were analyzed with a total of 277 curves. Seventy stations in problem areas* B-15, B-16, B-17, and B-18, were studied for peak discharge. Each group will be discussed with a tabular presentation of pertinent facts. Additional data are on file.

Table 1 represents, for each state concerned, the years of record and type distribution for the 314 non-recording 24-hour data. For these periods of record, only the classes 21-30 and 31-40 years are considered reliable for the development of synthetic data[2,3]. Distribution types Ln and An can be combined for comparison with the type E (extreme) distribution total. For this purpose, there are 140 of the combined Ln-An types and 174 of type E distribution. The author's proposed method could be used to evaluate 140 of the total 314 cases.

Table 2 presents the distribution types for various rainfall durations from the twenty-four recording stations. It is noteworthy that the type E distribution defined 143 curves, while types Ln and An defined 134. The one-hour duration appears to be better defined by the type E distribution on an average basis. The "period of record" for each of the 24 stations, the period 1903-1951, and other special periods comprise the 277 curves analyzed.

The distribution types are compared with classes of drainage area up to 1000 square miles in Table 3. Forty-seven type E distributions and 23 Ln-An types were noted. Fifty-two determinations were carried out on areas of less than 501 square miles drainage area.

Table 4 summarizes the information of Tables 1, 2, and 3. From this table, it can be seen that there are 364 type E distributions and 297 of type Ln and An. Approximately 45 percent of the 661 sets of data investigated could be "better" defined by the authors proposed method.

The writer has nothing but praise and admiration for the author's efforts in the field of engineering hydrology. If national, state and local agencies would publish data of this type in their files (closed files, etc.), and if some central organization could evaluate the procedures, the "battle" of man versus the elements would be won in less time and with less duplicated effort.

* Problem areas devised by the U. S. Soil Conservation Service (See Table 3.)

REFERENCES

1. Proceedings A.S.C.E., Separate No. 438, Vol. 80, May 1954.

2. Weather Bureau Technical Paper No. 24. "Rainfall Intensities for local drainage design in the United States." For durations of 5 to 240 minutes and 2-, 5-, and 10-year return periods. Part I west of the 115th meridian. Washington, D. C., 1953. Part II between 105°W. and 115°W., Washington, D. C., 1954.

3. Thesis to be submitted by William H. Sammons to the Graduate School of the University of Kentucky in partial fulfillment of the requirements for the degree of Master of Science in Civil Engineering.

Table 1. Years of Record and Distribution Type For Three-hundred and Fourteen Non-Recording Rainfall Stations in Ten States

State	Years of Record				State Totals	Distribution Type			Duration Totals
	0-10	11-20	21-30	31-40		Extreme Probability E	Logarithmic -Normal Ln	Arithmetic -Normal An	
Arkansas	-	3	1	30	34	16	10	8	34
Illinois	-	1	-	23	24	7	13	4	24
Indiana	-	3	2	16	21	11	9	1	21
Kentucky	3	20	31	34	88	60	20	8	88
Missouri	1	2	2	32	37	22	14	1	37
North Carolina	3	4	5	19	31	18	9	4	31
Ohio	-	1	2	16	19	11	6	2	19
Tennessee	-	4	3	17	24	10	10	4	24
Virginia	-	-	1	16	17	9	7	1	17
West Virginia	1	6	3	9	19	10	5	4	19
Years of Record and Distribution Type Totals	8	44	50	212	314	174	103 / 140	37	314

Table 2. Distribution Types For Twenty-Four Stations For Various Durations (Recording Rain Gauges)

Rainfall Duration	Distribution Type: Extreme Probability E	Logarithmic -Normal Ln	Arithmetic -Normal An	Duration Totals
5-minute	17	15	10	42
10-minute	17	9	18	44
15-minute	17	3	9	29
30-minute	19	8	4	31
1-hour	27	13	--	40
2-hour	16	9	1	26
24-hour	30	29	6	65
Distribution Type Totals	143	86 / 134	48	277

Table 4. Summary of Distribution Types

Type Data	Distribution Type: E	Ln	An	Total of Row
Non-Recording 24-hour Rainfall	174	103	37	314
24-hour Recording Rainfall	143	86	48	277
Peak Discharge	47	13	10	70
Distribution Type Totals	364	202 / 297	95	661

Table 3. Distribution Types For Seventy Stream Gauging Stations

(Problem Areas B-15; 16; 17; and 18*)

Drainage Area in square miles	Distribution Type			Drainage Area Size Totals
	E	Ln	An	
0 - 100	8	2	1	11
101 - 300	17	6	3	26
301 - 500	10	2	3	15
501 - 700	6	2	2	10
701 - 900	5	1	-	6
901 - 1000	1	-	1	2
Distribution Type Totals		13	10	
	47	23		70

* Map, "Problem Areas in Soil Conservation - Southeastern Region." U. S. Dept. of Agr., Soil Conservation Service, Agr. Soil Cons. Serv., Beltsville, Md., May 1951.

Discussion of
PROCEEDINGS-SEPARATES 361, 381, 466, 482, 483, 490

(Proc. Sep. 624)

Corrections to Discussion of Flood Insurance, by H. Alden Foster (Proceedings-Separate 483). The following illustration should have been included as a part of the discussion by Robert Lee Smith, page 624-26:

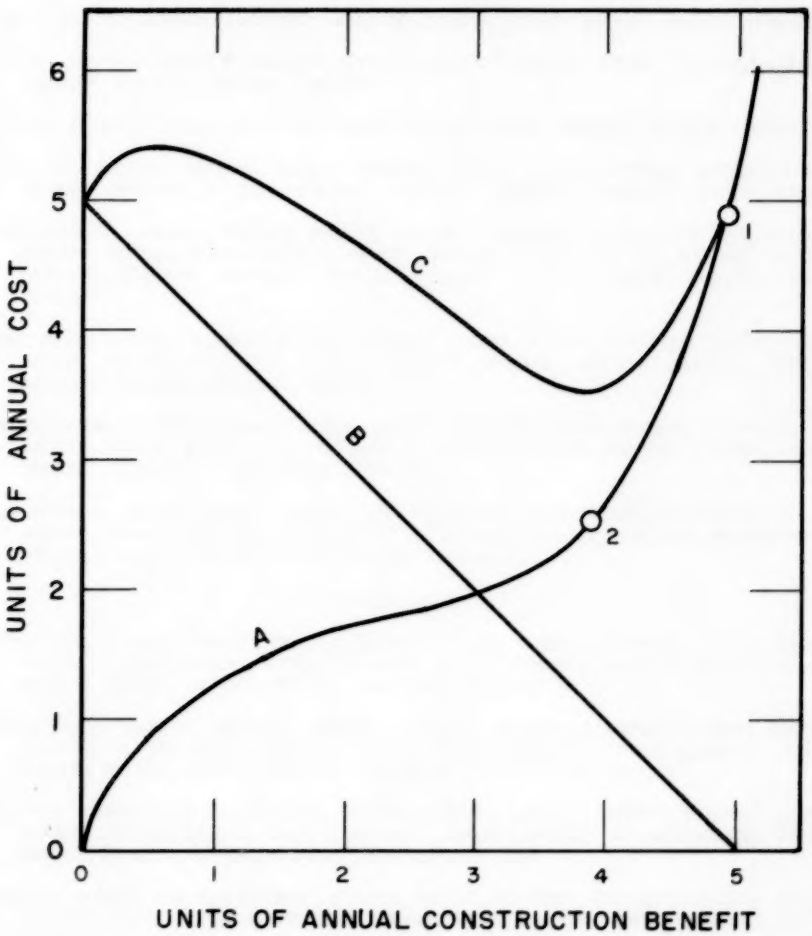


Figure 1.

PROCEEDINGS-SEPARATES

The technical papers published in the past year are presented below. Technical-division sponsorship is indicated by an abbreviation at the end of each Separate Number, the symbols referring to: Air Transport (AT), City Planning (CP), Construction (CO), Engineering Mechanics (EM), Highway (HW), Hydraulics (HY), Irrigation and Drainage (IR), Power (PO), Sanitary Engineering (SA), Soil Mechanics and Foundations (SM), Structural (ST), Surveying and Mapping (SU), and Waterways (WW) divisions. For titles and order coupons, refer to the appropriate issue of "Civil Engineering" or write for a cumulative price list.

VOLUME 80 (1954)

APRIL: 428(HY)[c], 429(EM)[c], 430(ST), 431(HY), 432(HY), 433(HY), 434(ST).

MAY: 435(SM), 436(CP)[c], 437(HY)[c], 438(HY), 439(HY), 440(ST), 441(ST), 442(SA), 443(SA).

JUNE: 444(SM)[e], 445(SM)[e], 446(ST)[e], 447(ST)[e], 448(ST)[e], 449(ST)[e], 450(ST)[e], 451(ST)[e], 452(SA)[e], 453(SA)[e], 454(SA)[e], 455(SA)[e], 456(SM)[e].

JULY: 457(AT), 458(AT), 459(AT)[c], 460(IR), 461(IR), 462(IR), 463(IR)[c], 464(PO), 465(PO)[c].

AUGUST: 466(HY), 467(HY), 468(ST), 469(ST), 470(ST), 471(SA), 472(SA), 473(SA), 474(SA), 475(SM), 476(SM), 477(SM), 478(SM)[c], 479(HY)[c], 480(ST)[c], 481(SA)[c], 482(HY), 483(HY).

SEPTEMBER: 484(ST), 485(ST), 486(ST), 487(CP)[c], 488(ST)[c], 489(HY), 490(HY), 491(HY)[c], 492(SA), 493(SA), 494(SA), 495(SA), 496(SA), 497(SA), 498(SA), 499(HW), 500(HW), 501(HW)[c], 502(WW), 503(WW), 504(WW)[c], 505(CO), 506(CO)[c], 507(CP), 508(CP), 509(CP), 510(CP), 511(CP).

OCTOBER: 512(SM), 513(SM), 514(SM), 515(SM), 516(SM), 517(PO), 518(SM)[c], 519(IR), 520(IR), 521(IR), 522(IR)[c], 523(AT)[c], 524(SU), 525(SU)[c], 526(EM), 527(EM), 528(EM), 529(EM), 530(EM)[c], 531(EM), 532(EM)[c], 533(PO).

NOVEMBER: 534(HY), 535(HY), 536(HY), 537(HY), 538(HY)[c], 539(ST), 540(ST), 541(ST), 542(ST), 543(ST), 544(ST), 545(SA), 546(SA), 547(SA), 548(SM), 549(SM), 550(SM), 551(SM), 552(SA), 553(SM)[c], 554(SA), 555(SA), 556(SA), 557(SA).

DECEMBER: 558(ST), 559(ST), 560(ST), 561(ST), 562(ST), 563(ST)[c], 564(HY), 565(HY), 566(HY), 567(HY), 568(HY)[c], 569(SM), 570(SM), 571(SM), 572(SM)[c], 573(SM)[c], 574(SU), 575(SU), 576(SU), 577(SU), 578(HY), 579(ST), 580(SU), 581(SU), 582(Index).

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JANUARY: 583(ST), 584(ST), 585(ST), 586(ST), 587(ST), 588(ST), 589(ST)[c], 590(SA), 591(SA), 592(SA), 593(SA), 594(SA), 595(SA)[c], 596(HW), 597(HW), 598(HW)[c], 599(CP), 600(CP), 601(CP), 602(CP), 603(CP), 604(EM), 605(EM), 606(EM)[c], 607(EM).

FEBRUARY: 608(WW), 609(WW), 610(WW), 611(WW), 612(WW), 613(WW), 614(WW), 615(WW), 616(WW), 617(IR), 618(IR), 619(IR), 620(IR), 621(IR)[c], 622(IR), 623(IR), 624(HY)[c], 625(HY), 626(HY), 627(HY), 628(HY), 629(HY), 630(HY), 631(HY), 632(CO), 633(CO).

MARCH: 634(PO), 635(PO), 636(PO), 637(PO), 638(PO), 639(PO), 640(PO), 641(PO)[c], 642(SA), 643(SA), 644(SA), 645(SA), 646(SA), 647(SA)[c], 648(ST), 649(ST), 650(ST), 651(ST), 652(ST), 653(ST), 654(ST)[c], 655(SA), 656(SM)[c], 657(SM)[c], 658(SM)[c].

APRIL: 659(ST), 660(ST), 661(ST)[c], 662(ST), 663(ST), 664(ST)[c], 665(HY)[c], 666(HY), 667(HY), 668(HY), 669(HY), 670(EM), 671(EM), 672(EM), 673(EM), 674(EM), 675(EM), 676(EM), 677(EM), 678(HY).

c. Discussion of several papers, grouped by Divisions.
e. Presented at the Atlantic City (N.J.) Convention in June, 1954.

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